# EFFECT OF VISCOSITY ON THE AUTOIGNITION OF A REACTING MOVING MIXTURE

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Ya. B. Zel'dovich has established [1] that in a continuous-flow reactor two ignition regimes are possible: forced ignition and autoignition.

It is important to consider the special properties of the autoignition regime associated with the hydromechanics of laminar flow and heat transfer through the pipe wall. In [2, 3] it was shown that the effect of heat of friction on heat transfer in long pipes is qualitative in character. Moreover, according to Schlichting [4], in certain cases the temperature gradient for such flows due to the heat of friction may reach 10–30°, which is comparable with the preexplosion temperature rise in the stationary theory of thermal explosion [5]. In this connection it is clear that under certain conditions the heat of friction may considerably reduce the explosion limit.

This paper is devoted to a study of the effect of heat of friction on the explosion limit of a reacting fluid in a long cylindrical pipe. The dynamic autoignition regime due to heat of friction is examined. In particular, it is established that, other things being equal, by increasing the pressure drop it is possible to obtain explosion of the reacting system.

**§1.** The laminar steady-state flow of a viscous incompressible fluid with temperature-dependent viscosity in a semi-infinite circular pipe is described [6, 7] by the system of equations of motion and conservation of energy. In our case it is necessary to supplement the latter equation with a term characterizing the heat due to the chemical reaction, so that the system of equations takes the form

$$\frac{d}{dr}\left[r\mu(T)\frac{dw}{dr}\right] = r\frac{dp}{dz}, \qquad (1.1)$$

$$\lambda\frac{d}{dr}\left(r\frac{dT}{dr}\right) + qk_0 r \exp\frac{-E}{RT} + \frac{\mu(T)}{J}r\left(\frac{dw}{dr}\right)^2 = 0. \qquad (1.2)$$

Here, w is the flow velocity, T the absolute temperature, dp/dz the pressure drop along the pipe, r the present radius, $\lambda$ the thermal conductivity, J the mechanical equivalent of heat, q the reaction energy per unit volume, R the universal gas constant, E the activation energy, and $k_0$ the preexponential factor.

The boundary conditions for system (1.1), (1.2) have the form

$$dT/dr|_{r=0} = 0, \quad T(r_0) = T_0,$$
$$dw/dr|_{r=0} = 0, \qquad w(r_0) = 0. \qquad (1.3)$$

We assume that the viscosity depends on temperature in the following way [7]:

$$\mu = \mu_0 \exp(E_1/RT)$$
$$(\mu_0 = \text{const}, \ E_1 = \text{const}). \qquad (1.4)$$

Eliminating w(r) from system (1.1), (1.2), using the Filonov [7] approximation for $\mu$(T) and the Frank-Kamenetskii approximation for the chemical reaction rate, and reducing the equation obtained to dimensionless form, we obtain

$$\frac{1}{y}\frac{d}{dy}\left(y\frac{d\theta}{dy}\right) + \delta e^{\theta} + \beta\delta^2 y^2 e^{b\theta}$$
$$\left(\delta = \frac{qk_0 r_0^2 E}{\lambda R T_0^2}\exp - \frac{E}{RT}\right) \qquad (1.5)$$

with boundary conditions

$$d\theta/dx|_{x=0} = 0, \qquad \theta(1) = 0,$$
$$\beta = \frac{\lambda R T_0^2}{4J\mu_0 q^2 k_0^2}\left(\frac{dp}{dz}\right)^2 \exp\frac{2E - E_1}{RT_0},$$
$$\left(\theta = \frac{(T - T_0)E}{RT_0^2}\right). \qquad (1.6)$$

Here, $\theta$ is the dimensionless temperature, $\beta$ is a dimensionless parameter characterizing the intensity of the mechanical heat sources due to dissipation of the kinetic energy of the flow, $\delta$ is the Frank-Kamenetskii number [5], $y = r/r_0$ is a dimensionless coordinate, $b = E_1/E$ is a parameter characterizing the degree of dependence of the mechanical heat sources on temperature, usually $b < 1$.

The boundary-value problem (1.5), (1.6) does not have a solution at all values of $\delta$. We will call the limiting value $\delta = \delta_*$, at which a real solution of the problem still exists, the explosion limit. The quantity $\delta_*$ is proportional to $r_0^2$; therefore the problem of determining this limit may be formulated as follows: for a given pipe wall temperature and specified pressure drop along the pipe determine the pipe radius corresponding to autoignition of the reacting mixture.

**§2.** Making the substitution $u = \theta - \theta_0$, where $\theta_0 = \theta(0)$, by successive integration of Eq. (1.5) we reduce boundary-value problem (1.5), (1.6) to the nonlinear Volterra integral equation

$$u = -8m\int_0^y\left(x^{-1}\int_0^x \xi e^{u(\xi)}d\xi\right)dx -$$
$$-\beta\delta^2 e^{b\theta_0}\int_0^y\left(x^{-1}\int_0^x \xi^3 e^{bu(\xi)}d\xi\right)dx \qquad \left(m = \frac{\delta\exp\theta_0}{8}\right). \qquad (2.1)$$

If we substitute for u in the right side of Eq. (2.1) a value that we know to be too high, for example, $u_0^+ \equiv 0$, we obviously obtain a function $u_1^-(y)$ which on the interval $0 < y \le 1$ is smaller than u, the true solution of Eqs. (2.1). Substituting $u_1^-(y)$ into the right side of (2.1), we obtain $u_2^+(y) > u(y)$. Obviously, $u_2^+(y) < 0$. Substituting the second approximation $u_2^+$ for u in the right side of (2.1), we obtain $u_3^- < u$, but at the same time $u_3^- > u_1^-$, since $u_2^+ < 0$. Substituting the third approximation for u in (2.1), we obtain $u_4^+ > u$, but at the same time $u_4^+ < u_2^+$, since $u_3^- > u_1^-$, and so on.

Thus, we have obtained a sequence of upper functions $u_0^+ > u_2^+ > \cdots > u$ and a sequence of lower functions $u_1^- < u_3^- < \ldots < u$. Since the sequence of upper functions $\{u_{2i}^+\}$ decreases and is bounded below by the true solution u, it converges to u. The sequence of lower function $\{u_{2i+1}^-\}$ also converges to u, since it increases and is bounded above by the true solution u.

Thus, the solution of Eq. (2.1) can be found with any degree of accuracy, and in each step of the calculations it is possible to determine the error of the approximate solution, for which it is sufficient to determine the difference $u_{2i}^+ - u_{2i+1}^-$. If this difference is small at $0 < y \leq 1$, then as the approximate value of u it is possible to take $u_{2i}^+$ or $u_{2i+1}^-$. The convergence of the successive approximations can also be demonstrated with the help of [8]. Having determined $u_{i-1} \approx u$, we satisfy the second of boundary conditions (1.6). In this case we obtain an equation giving $\delta_i$ as a function of $\theta_{0i}$:

$$\theta_{0i} + 8m_i \int_0^1 x \ln x e^{u_{i-1}(x)} dx + \beta\delta_i^2 e^{b\theta_{0i}} \int_0^1 x^3 \ln x e^{bu_{i-1}(x)} dx = 0. \quad (2.2)$$

It turns out that $\delta_i(\theta_{0i})$ is nonmonotonic, and at $\theta_{0i} = \theta_{0i*}$ the quantity $\delta_i$ has a maximum $\delta_{i*}$. Differentiating (2.2) with respect to $\theta_{0i}$, provided that $d\delta_i \neq d\theta_{0i}$ we obtain the equation

$$1 + 8m_i \int_0^1 x \ln x \left(1 + \frac{\partial u_{i-1}}{\partial \theta_{0i}}\right) e^{u_{i-1}(x)} dx + \beta\delta_i^2 b e^{b\theta_{0i}} \times \int_0^1 x^3 \ln x \left(1 + \frac{\partial u_{i-1}}{\partial \theta_{0i}}\right) e^{bu_{i-1}(x)} dx = 0. \quad (2.3)$$

System of equations (2.2), (2.3) determines the quantities $\delta_{i*}$ and $\theta_{0i*}$, which represent approximations of the maximum temperature rise and the explosion limit.

The effectiveness of the method is apparent from simple examples of the autoignition of reacting plates, cylinders, and spheres, for which exact solutions are known [5]. Thus, for the autoignition of a plate $\delta_{1*} = 0.74, \theta_{01*} = 1$ and $\delta_{2*} = 0.90$, $\theta_{02*} = 1.22$, whereas the exact values [5] are $\delta_* = 0.88$, $\theta_{0*} = 1.2$.

§3. We will first consider the autoignition of a reacting mixture at constant viscosity (b = 0). In accordance with [3, 4], this case is realized for certain liquids, as well as for any gases provided that the flow velocity is small as compared with the speed of sound.

For the autoignition of a reacting mixture in a tube it is convenient to take as the zero-order approximation of the solution of (2.1) the function

$$u_0^+ = -2 \ln(1 + my^2), \quad (3.1)$$

which is the solution of (2.1) at $\beta = 0$. This function can be found with the help of [9]. Obviously, $u_0^+ > u$, where u— is the solution of Eq. (2.1). Substituting $u_0^+$ into the right-hand side of Eq. (2.1), we obtain the first approximation

$$u_1^- = {}^1/_{12}\beta\delta^2 y^4 - 2\ln(1 + my^2), \quad (3.2)$$

which, obviously, is smaller than u. Substituting this expression into the right side of (2.1), we obtain the second approximation

$$u_2^+ = -\frac{\beta\delta^2 y^4}{16} - 8m \int_0^y x \ln\frac{y}{x} (1 + mx^2)^{-2} \exp\frac{-\beta\delta^2 x^4}{16} dx \quad (3.3)$$

known to be too high as compared with u.

Substituting (3.1) into system (2.2), (2.3), we obtain the following system of equations for determining $\delta_{1*}$ and $\theta_{01*}$:

$$\theta_{01*} = 2\ln 2 + 4\beta \exp(-2\theta_{01*}), \quad \delta_{1*} = 8 \exp(-\theta_{01*}). \quad (3.4)$$

Solutions of system (3.4) for a number of values of $\beta$ are given in the table:

| $\beta$ | 0.01 | 0.1 | 1 | 10 | 100 |
|---|---|---|---|---|---|
| $\delta_{1*}$ | 1.99503 | 1.9531 | 1.6775 | 1.0303 | 0.4784 |
| $\theta_{01*}$ | 1.38878 | 1.4100 | 1.5622 | 2.0496 | 2.8167 |
| $\delta_{2*}$ | 1.99548 | 1.9578 | 1.7006 | 1.0581 | 0.4939 |
| $\theta_{02*}$ | 1.38884 | 1.4103 | 1.5628 | 2.0648 | 2.8752 |

Substituting (3.2) into system of equations (2.2), (2.3), we obtain a system of equations for determining $\delta_{2*}$ and $\theta_{02*}$. This system has been solved by Newton's method [10], taking as the first approximation the corresponding values $\delta_{1*}$ and $\theta_{01*}$, and evaluating the definite integrals in the system of equations for determining $\delta_{2*}, \theta_{02*}$ by Simpson's method for 20 ordinates [10] using the tables given in [11]. The results of the calculations are presented in the table.

From the data presented above it is clear that the difference between the first and second approximations, while remaining quite small, increases with $\beta$, which is perfectly legitimate since the zero-order approximation is exact at $\beta = 0$. If we assume, by analogy with the example of §2, that the first approximation of $\delta_*$ and $\theta_{0*}$ gives too low a value of these quantities, while the second approximation $\delta_{2*}$ and $\theta_{02*}$ gives too high a value of $\delta_*$ and $\theta_{0*}$, then the small value of the differences $\delta_{2*} - \delta_{1*}$, $\theta_{02*} - \theta_{01*}$ indicates that in practice the second approximation of $\delta_*$ and $\theta_{0*}$ may be regarded as the exact value of those quantities. In order to establish whether $\delta_{1*}, \theta_{01*}$ and $\delta_{2*}, \theta_{02*}$ are respectively the lower and upper bounds of $\delta_*$ and $\theta_{0*}$, we calculated $\delta_{3*}, \theta_{03*}$ for $\beta = 100$. For this purpose we substituted expression (3.3) for u in system of equations (2.2), (2.3). The system of equations obtained gives $\delta_3$ and $\theta_{03*}$. This system was solved by Newton's method, and as the zero-order approximation we took the quantities $\delta_{2*}$ and $\theta_{02*}$ for $\beta = 100$. The definite integrals in the system of equations for determining $\delta_{3*}$ and $\theta_{03*}$ were evaluated by Simpson's method for 20 ordinates, while the integrals with a variable upper limit were determined by Melent'ev's method [10] for four ordinates with a step h = 0.05. As a result of the calculation we found $\delta_{3*} = 0.4920$, $\theta_{03*} = 2.8654$. These values fall between the values of $\delta_{1*}, \theta_{01*}$ and $\delta_{2*}, \theta_{02*}$ and lie closer to the latter, as was to be expected.

§4. We will consider the autoignition of a reacting mixture at b = 1/2. Selecting, as before, (3.1) as the zero-order approximation, we obtain the analogous system of equations

$$\theta_{01} - 2\ln(1 + m_1) - 2\beta\delta_1 e^{-1/2\,\theta_{01}} + 32\beta e^{-3/2\,\theta_{01}} \int_0^1 \frac{\ln(1 + m_1x^2)\,dx}{x} = 0, \quad (4.1)$$

$$1 - \frac{2m_1}{1 + m_1} + \beta\delta_1 e^{-1/2\,\theta_{01}} - 48\beta e^{-3/2\,\theta_{01}} \int_0^1 \frac{\ln(1 + m_1x^2)\,dx}{x} + 16\beta e^{-3/2\,\theta_{01}} \ln(1 + m_1) = 0 \quad (4.2)$$

for the quantities $\delta_{1*}$ and $\theta_{01*}$. This system of equations was solved by Newton's method [10]. The definite integral in system (4.1), (4.2) was evaluated for $m_1 = 1$ using the tables given in [12], for $m_1 = 1 - \varepsilon$, where $\varepsilon \ll 1$,

$$\int_0^1 \frac{\ln(1 + m_1x^2)\,dx}{x} = \frac{\pi^2}{24} - 0.346573\,\varepsilon - 0.048287\,\varepsilon^2 - 0.003797\,\varepsilon^3 - \ldots. \quad (4.3)$$

Calculations at $\beta$ = 0.001, 0.01, 0.1, 1 yield $\delta_{1*}$ = 1.999290, 1.9929, 1.93, 1.62 and $\theta_{01*}$ = 1.386487, 1.3881, 1.40, 1.44.

The evaluation of the second approximation involves considerable computation; therefore we will check the accuracy of the quantities $\delta_{1*}$ and $\theta_{01*}$ for small $\beta$ by means of the small parameter method. We write the solution of Eq. (2.1) for small $\beta$ in the form

$$u = -2\ln(1+my^2)+\beta u_1+\beta^2 u_2+\ldots \quad (4.4)$$

Substituting (4.4) into (2.1) and discarding small quantities of the second order and above, we obtain an equation for $u_1$, solving which we find

$$u_1 = \delta^2 e^{1/2\theta_0}\left[\frac{y^2(9-my^2)}{4m(1+my^2)}-\frac{3\ln(1+my^2)}{2m^2}-\right.$$
$$\left.-\frac{3(1-my^2)}{2m^2(1+my^2)}\int_0^y\frac{\ln(1+my^2)\,dy}{y}\right]. \quad (4.5)$$

Satisfying (4.4) with account for (4.5) and the second of conditions (1.6), we obtain an equation giving $\delta$ as a function of $\theta_0$:

$$\theta_0-2\ln(1+m)+\beta\delta^2 e^{1/2\theta_0}\left[\frac{9-m}{4m(1+m)^2}-\frac{3\ln(1+m)}{2m^2}-\right.$$
$$\left.-\frac{3(1-m)}{2m^2(1+m)}\int_0^1\frac{\ln(1+mx^2)\,dx}{x}\right]=0. \quad (4.6)$$

Differentiating (4.5) with respect to $\theta_0$ and noting that $d\delta/d\theta_0 = 0$, we have

$$1-\frac{2m}{1+m}+\frac{\beta\delta^2}{2}e^{1/2\theta_0}\left[\frac{m^2-40m-21}{4m(1+m)^2}+\right.$$
$$+\frac{3(1+2m)\ln(1+m)}{m^2(1+m)}+$$
$$\left.+\frac{3(3-3m^2+4m)}{2m^2(1+m)^2}\int_0^1\frac{\ln(1+mx^2)\,dx}{x}\right]=0. \quad (4.7)$$

Equations (4.6) and (4.7) determine $\delta_*$ and $\theta_{0*}$ correct to terms containing $\beta^2$, so that at small $\beta$ the quantities $\delta_*$ and $\theta_{0*}$, determined from system of equations (4.6) and (4.7), must be close to the exact values. System of equations (4.6), (4.7) was solved by Newton's method [10]. For $\beta$ = 0.001, 0.01, 0.1 we obtained $\delta_*$ = 1.995365, 1.9937, 1.94, $\theta_{0*}$ = 1.386500, 1.3883, 1.40, respectively. Comparing these data with the data previously obtained by the method of successive approximations, we see that $\delta_{1*}$ and $\theta_{01*}$ approximate $\delta_*$ and $\theta_{0*}$ from below, and the error is not large.

From the tabulated data and the above calculations it follows that the preexplosion temperature rise $\theta_{0*}$ increased with $\beta$, while the explosion limit decreases. Physically, this is attributable to the fact that the heat of friction causes a local increase in temperature near the wall, which is greater, the greater $\beta$, as a result of which the flow of heat from the central part of the pipe is reduced more strongly, the greater $\beta$.

Comparing the tabulated data and the data of the above calculations, we see that the values of $\delta_*$ and $\theta_{0*}$ for $b \neq 0$ are lower than the corresponding values of $\delta_*$ and $\theta_{0*}$ for $b = 0$. This is attributable to the fact that the amount of heat derived from mechanical heat sources is greater at $b \neq 0$ than at $b = 0$, while the decrease in $\theta_{0*}$ at $b \neq 0$ is attributable to the fact that the temperature due to mechanical heat sources rises more uniformly at $b \neq 0$ than at $b = 0$, as a result of which the flow of heat from the central part of the pipe increases. In view of the symmetry conditions, the temperature maximum is reached at $y = 0$ in both cases.

§5. We will estimate the effect of the heat of friction on the autoignition of a reacting fluid for Newtonian heat transfer through the pipe wall. For this purpose we will consider the thermal explosion of a reacting fluid initially at rest in an infinite cylindrical pipe and then suddenly brought into motion. This problem is an example of dynamic autoignition, different from the examples examined in [13–14]. Mathematically the problem reduces to the solution of the system of equations

$$\frac{\partial w}{\partial t}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)-\frac{1}{\rho}\frac{dp}{dz}, \quad (5.1)$$

$$c_p\rho\frac{\partial T}{\partial t}=\frac{\lambda}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+qk_0\exp\frac{-E}{RT}+\mu\left(\frac{\partial w}{\partial r}\right)^2 \quad (5.2)$$

with boundary and initial conditions

$$\left.\frac{\partial w}{\partial r}\right|_{r=0}=0,\quad w(t,r_0)=0,\quad \left.\frac{\partial T}{\partial r}\right|_{r=0}=0,$$
$$T(t,r_0)=T_0,\qquad T(0,r)=T_0,\qquad w(0,r)=0. \quad (5.3)$$

Here, t is time, $c_p$ the specific heat at constant pressure, $\rho$ density, and $\nu$ the kinematic viscosity.

For simplicity we assume that the viscosity and the thermophysical coefficients are constant.

An exact solution of Eq. (5.1) with conditions (5.3) has been obtained by Gromeko [6] in the form of a series in Bessel functions. To simplify the subsequent analysis, we will use the method of integral relations [15] to find a simple approximate solution of Eqs. (5.1) with conditions (5.3):

$$w=-\frac{r_0^2}{4\mu}\frac{dp}{dz}\left(1-\exp\left(-\frac{8\nu t}{r_0^2}\right)\right)\left(1-\frac{r^2}{r_0^2}\right). \quad (5.4)$$

To derive (5.4), the profile $w = w_0(t)(1 - r^2/r_0^2)$ is substituted into Eq. (5.1), the result of the substitution is integrated with respect to r from 0 to $r_0$ and the first-order differential equation for $w_0(t)$ thus obtained is solved with the zero condition. A comparison of (5.4) with the exact solution showed that the error of (5.4) does not exceed 12%.

Substituting (5.4) into (5.2) and reducing the result of the substitution to dimensionless form, we have the equation

$$y\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial y}\left(y\frac{\partial\theta}{\partial y}\right)+\delta y e^{\theta}+\beta\delta^2 y^3(1-e^{-8P\tau})^2$$
$$\left(\tau=\frac{\lambda t}{\rho c_p r_0^2}\right) \quad (5.5)$$

with boundary and initial conditions

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0}=0,\quad \left.\left(\frac{\partial\theta}{\partial y}+B\theta\right)\right|_{y=1}=0,\quad \theta(0,y)=0$$
$$\left(B=\frac{\alpha r_0}{\lambda},\quad P=\frac{\nu\rho c_p}{\lambda}\right). \quad (5.6)$$

Here, P is the Prandtl number and $\alpha$ the heat transfer coefficient.

We employ the method of integral relations [14,15] to solve boundary-value problem (5.5), (5.6). We assume that the temperature profile [14] has the form

$$f=g(\tau)-2\ln(1+ay^2)$$
$$(g=2\ln(1+a)+4a/B(1+a)). \quad (5.7)$$

Substituting (5.7) into (5.5) and integrating the result of the substitution with respect to y from 0 to 1, we obtain the Cauchy problem for determining $a(\tau)$:

$$\frac{da}{d\tau}=\left[Ba^2(1+a)\{\delta(1+a)[2e^{4a/B(1+a)}+\right.$$
$$\left.+\beta\delta(1-e^{-8P\tau})^2]-16a\}\right]\left[4\{a^2[2+B(1+a)]-\right.$$
$$\left.-B(1+a)^2[a-\ln(1+a)]\}\right]^{-1},\quad a(0)=0. \quad (5.8)$$

If $a(\tau) \to \infty$ as $\tau \to \tau_0 < \infty$, i.e., if the solution of the Cauchy problem (5.8) has finite determination time [16], then the reacting system will explode, and in this case the quantity $\tau_0$ is the induction period. Taking $\tau$ as the function, and $a$ as the independent variable, we easily find that an explosion will occur if as $a \to \infty$ we have $\tau(a) \to \tau_0 < \infty$, i.e., in this case the problem of thermal explosion reduces to the Lagrange stability [16] for $\tau = \tau(a)$.

We will show that for any $\beta$ there exists an explosion limit $\delta = \delta_*$. The function $a^+(\tau)$ given by the equation

$$\frac{da^+}{d\tau} = \Big[Ba^{+2}(1+a^+)\{\delta(1+a^+)\times \times\{\beta\delta + 2\exp[4a^+/B(1+a^+)]\} - 16a^+\}\Big]\times \times\Big[4\{a^{+2}[2+B(1+a^+)] - B(1+a^+)^2[a^+ - \ln(1+a^+)]\}\Big]^{-1} \qquad (5.9)$$

with initial condition (5.8) majorizes $a = a(\tau)$. The solution of the Cauchy problem (5.8), (5.9) for $\delta \le \delta_*^-$ and $\tau \to \infty$ takes finite stationary values, and at $\delta > \delta_*^-$ there is a steady increase in the quantity $a^+$ with increase in $\tau$, so that $\lim a^+(\tau) = \infty$ as $\tau \to \tau_0$.

It is easy to see that the limiting value of $\delta = \delta_*^-$, at which the stationary value of $a^+$ is reached, and the corresponding value $a_*$ are determined by the system of equations

$$\frac{B}{4+B(1+a)}\left[1 + \frac{4\beta B}{(1+a)^2[4+B(1+a)]}\times \times\exp\frac{-8a}{B(1+a)}\right] = \frac{a}{1+a},$$

$$\delta = \frac{8B}{(1+a)[4+B(1+a)]}\exp\frac{-4a}{B(1+a)}. \qquad (5.10)$$

As $B \to \infty$ system (5.10) reduces to the single equation

$$\delta = 2\,(1 - {}^1/_{16}\,\beta\delta^2)^2 \qquad (5.11)$$

whose solution

$$\delta_*^- = 2\{1 - {}^1/_4\beta\,[1 - {}^1/_4\beta\,(1 - {}^1/_4\beta)^4]^2\}, \qquad (5.12)$$

which we found by an iterative method [10], satisfactorily coincides with the tabulated data at $0 < \beta \le 1$. When $\beta \gg 1$ we have $a_* \gg 1$ and $\delta_*^- \approx 4/(\beta)^{1/2}$.

Using a small parameter method [10], for small values of B we found

$$\delta_*^- = \frac{2B}{e}\left[1 - \frac{3B}{4}\left(1 + \frac{4\beta}{3e^2}\right)\right]. \qquad (5.13)$$

Since $a < a^+$, it is clear that when $\delta \le \delta_*^-$ and $\tau \to \infty$ the quantity $a \to$ const $< \infty$. At the same time, in the absence of friction ($\beta = 0$) a stationary temperature distribution exists at $\delta \le \delta_*$, where

$$\delta_*^+ = \frac{8a_*}{(1+a_*)^2}\exp\frac{-4a_*}{B(1+a_*)}$$

$$\left(a_* = 2\frac{1}{B}\left(\sqrt{1 + \frac{B^2}{4}} - 1\right)\right). \qquad (5.14)$$

Expression (5.14) is easily obtained from system (5.10) and coincides with the corresponding exact value of the explosion limit [17].

Thus, in the case considered when $\beta \ne 0$ an explosion limit always exists and lies in the range $\delta_*^- \le \delta_* \le \delta_*^+$.

It is interesting to note that if a dynamic autoignition regime is achieved by raising the external temperature, assuming [14] that as $\tau \to \tau_1 < \tau_0$ it increases smoothly from 0 to $\Theta_{01}$, then, as distinct from the case considered, at $\Theta_{01} > 2\ln 2$ there will be an unstable temperature distribution and an explosion will follow the least perturbation at any value of $\delta > 0$.

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